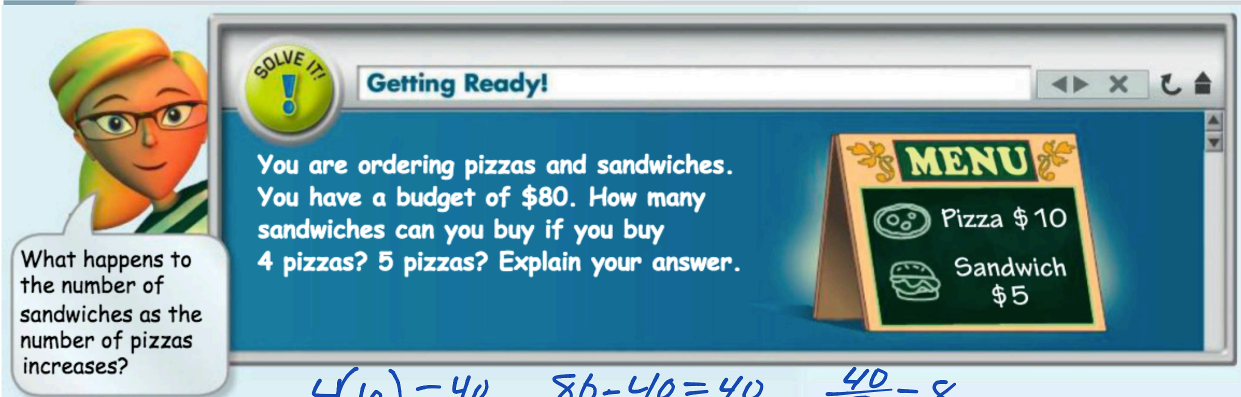
# 2-5 Literal Equations and Formulas




$5(10)=50$

$80-50=30$

$\frac{30}{5}=6$

$4(10)=40 \quad 80-40=40 \quad \frac{40}{5}=8$

In this lesson, you will learn to solve problems using equations in more than one variable. A ***literal equation*** *is an equation that involves two or more variables.* When you work with literal equations, you can use the methods you have learned in this chapter to isolate any particular variable.

**PROBLEM 1: REWRITING A LITERAL EQUATION**

a) The equation $10x + 5y = 80$, where *x* is the number of pizzas and *y* is the number of sandwiches, models the problem in Solve It. How many sandwiches can you buy if you buy 3 pizzas? 6 pizzas?

$10x+5y=80$
$-10x \qquad -10x$

$\frac{5y}{5} = \frac{80-10x}{5}$

$y = \frac{80}{5} - \frac{10x}{5}$

$\boxed{y=16-2x}$

$x=3;\ y=16-2(3)$
$y=16-6$
$y=10$

10 Sandwiches

$x=6;\ y=16-2(6)$
$y=16-12$
$y=4$

4 Sandwiches

b) Solve the equation $4 = 2m - 5n$ for $m$. What are the values of *m* when n = -2, 0, and 2.

$+5n \qquad +5n$

$\frac{4+5n}{2} = \frac{2m}{2}$

$\boxed{\frac{4+5n}{2} = m}$

$n=-2;\ m=\frac{4+5(-2)}{2}$
$m=\frac{4-10}{2}$
$m=\frac{-6}{2}$
$m=-3$

$n=0;\ m=\frac{4+5(0)}{2}$
$m=\frac{4+0}{2}$
$m=\frac{4}{2}$
$m=2$

$n=2;\ m=\frac{4+5(2)}{2}$
$m=\frac{4+10}{2}$
$m=\frac{14}{2}$
$m=7$

$(-3,-2)$

*Solve each equation for y. Then find the value of y for each value of x.*

c) $2y + 7x = 4; \boxed{x = 5, 10, 15}$

$-7x \quad -7x$

$\frac{2y}{2} = \frac{4-7x}{2}$

$\boxed{y = \frac{4-7x}{2}}$

$x=5;\ y = \frac{4-7(5)}{2}$, $y = \frac{4-35}{2}$, $y = -\frac{31}{2}$

$x=15;\ y = \frac{4-7(15)}{2}$, $y = \frac{4-105}{2}$, $y = \frac{-101}{2}$

$2y + 50 = 4$, $-50 \quad -50$, $\frac{2y}{2} = \frac{4-50}{2}$, $y = \frac{4-50}{2}$

$x=10;\ y = \frac{4-7(10)}{2}$, $y = \frac{4-70}{2}$, $y = -\frac{66}{2}$, $y = -33$

d) $6x = 7 - 4y; x = -2, -1, 0$

$-7 \quad -7$

$\frac{6x-7}{-4} = \frac{-4y}{-4}$

$\frac{6x-7}{-4} = y$

or $-\frac{6x-7}{4} = y$

or $\boxed{\frac{7-6x}{4} = y}$

$\boxed{-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}}$

$x=-2;\ y = \frac{7-6(-2)}{4}$, $y = \frac{7+12}{4}$, $y = \frac{19}{4}$

$x=-1;\ y = \frac{7-6(-1)}{4}$, $y = \frac{7+6}{4}$, $y = \frac{13}{4}$

$x=0;\ y = \frac{7-6(0)}{4}$, $y = \frac{7-0}{4}$, $y = \frac{7}{4}$

When you rewrite literal equations, you may have to divide by a variable or variable expression. When you do so in this lesson, assume that the variable or variable expression is not equal to zero because division by zero is not defined.

**PROBLEM 2: REWRITING A LITERAL EQUATION WITH ONLY VARIABLES**

*Solve each equation for x.*

$5x - 2x$
$(5-2)x$

a) $ax - bx = c$

$\frac{\cancel{(a-b)}x}{\cancel{(a-b)}} = \frac{c}{(a-b)}$

$\boxed{x = \frac{c}{a-b}}$

b). $-t = r + px$

$-r \quad -r$

$\frac{-t-r}{p} = \frac{px}{p}$

$\boxed{-\frac{t+r}{p} = x}$

c) $A = Bxt + C$

$-C \quad -C$

$\frac{A-C}{Bt} = \frac{\cancel{Bxt}}{\cancel{Bt}}$

$\boxed{\frac{A-C}{Bt} = x}$

d) $\cancel{(y-1)}\left(\frac{x+2}{\cancel{y-1}}\right) = 2(y-1)$

$x + 2 = 2y - 2$

$-2 \quad -2$

$\boxed{x = 2y - 4}$

e) $mx + nx = p$

$\frac{\cancel{(m+n)}x}{\cancel{(m+n)}} = \frac{p}{(m+n)}$

$\boxed{x = \frac{p}{m+n}}$

f) $b(y) = \left(\frac{x-v}{\cancel{b}}\right)\cancel{b}$

$by = x - v$

$+v \quad +v$

$\boxed{by + v = x}$

g) $4(x - b) = x$

$4x - 4b = x$

$-x \quad -x$

$3x - 4b = 0$

$+4b \quad +4b$

$\frac{3x}{3} = \frac{4b}{3}$

$\boxed{x = \frac{4b}{3}}$

or

$\boxed{x = \frac{4}{3}b}$

h) $\cancel{a}\left(\frac{x}{\cancel{a}}\right) = \left(\frac{y}{b}\right)\frac{a}{1}$

$\boxed{x = \frac{ay}{b}}$

A ***formula*** *is an equation that states a relationship among quantities.* Formulas are special types of literal equations. Some common formulas are given below. Notice that some of the formulas use the same variables, but the definitions of the variables are different.

| Formula Name | Formula | Definitions of Variables |
|---|---|---|
| Perimeter of a rectangle | $P = 2\ell + 2w$ | $P$ = perimeter, $\ell$ = length, $w$ = width |
| Circumference of a circle | $C = 2\pi r$ | $C$ = circumference, $r$ = radius |
| Area of a rectangle | $A = \ell w$ | $A$ = area, $\ell$ = length, $w$ = width |
| Area of a triangle | $A = \frac{1}{2}bh$ | $A$ = area, $b$ = base, $h$ = height |
| Area of a circle | $A = \pi r^2$ | $A$ = area, $r$ = radius |
| Distance traveled | $d = rt$ | $d$ = distance, $r$ = rate, $t$ = time |
| Temperature | $C = \frac{5}{9}(F - 32)$ | $C$ = degrees Celsius, $F$ = degrees Fahrenheit |

**PROBLEM 3: REWRITING A GEOMETRIC FORMULA**

*Solve each problem. Round to the nearest tenth, if necessary. Use 3.14 for π.*

a) What is the radius of a circle with circumference 22 m?

$\frac{C}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$ $\quad C=22;$ $\quad \frac{22}{2\pi} = r$ $\quad$ ~~22÷2·3.14~~

$\boxed{\frac{C}{2\pi} = r}$ $\quad 3.5 = r$ $\quad 22 \div (2 \cdot 3.14)$

3.5 m

b) What is the height of a triangle that has an area of 24 in$^2$ and a base with a length of 8 in?

$2(A) = (\tfrac{1}{\cancel{2}} bh)\cancel{2}$ $\quad A=24, b=8;$ $\quad \frac{2(24)}{8} = h$

$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$ $\quad 6 = h$

$\boxed{\frac{2A}{b} = h}$

6 in

c) What is the length of a rectangle with width 19 in and area 45 in$^2$?

$\frac{A}{w} = \frac{\ell \cancel{w}}{\cancel{w}}$ $\quad w=19, A=45;$ $\quad \frac{45}{19} = \ell$

$\boxed{\frac{A}{w} = \ell}$ $\quad 2.4 = \ell$

2.4 in

d) A triangle has height 4 ft and area 32 ft$^2$. What is the length of its base?

$2(A) = (\tfrac{1}{\cancel{2}} bh)\cancel{2}$ $\quad h=4, A=32;$ $\quad \frac{2(32)}{4} = b$

$\frac{2A}{h} = \frac{bh}{h}$ $\quad 16 = b$

$\boxed{\frac{2A}{h} = b}$

16 ft

e) A rectangle has perimeter 84 cm and length 35 cm. What is its width?

$P = 2\ell + 2w$ $\quad P=84, \ell=35;$ $\quad \frac{84 - 2(35)}{2} = w$

$-2\ell \quad -2\ell$

$\frac{P - 2\ell}{2} = \frac{\cancel{2}w}{\cancel{2}}$ $\quad \frac{84-70}{2} = w$

$\boxed{\frac{P-2\ell}{2} = w}$ $\quad \frac{14}{2} = w$

$7 = w$

7 cm

**PROBLEM 4: REWRITING A FORMULA**

a) The monarch butterfly is the only butterfly that migrates annually north and south. A particular group of monarch butterflies travels a distance of 1700 miles. It takes a typical butterfly about 120 days to travel one way. What is the average rate at which a butterfly travels in miles per day? Round to the nearest mile per day.

$$\frac{d}{t} = \frac{r\cancel{t}}{\cancel{t}}$$

$$\boxed{\frac{d}{t} = r}$$

$$r = \frac{1700}{120}$$

$$r \approx 14$$

14 miles/day

b) You can use the formula $a = \frac{h}{n}$ to find the batting average *a* of a batter who has *h* hits in *n* times at bat. Solve the formula for *h.* If a batter has a batting average of .290 and has been at bat 300 times, how many hits does the batter have?

$$n(a) = \left(\frac{h}{\cancel{n}}\right)\cancel{n}$$

$$\boxed{an = h}$$

$$h = .290(300)$$

$$h = 87$$

87 hits

c) Bricklayers use the formula $n = 7lh$ to estimate the number *n* of bricks needed to build a wall of length *l* and height *h*, where *l* and *h* are in feet. Solve the formula for *h.* Estimate the height of a wall 28 ft long that requires 1568 bricks.

$$\frac{n}{7l} = \frac{\cancel{7l}h}{\cancel{7l}}$$

$$\boxed{\frac{n}{7l} = h}$$

$$h = \frac{1568}{7(28)}$$

$$h = \frac{1568}{196}$$

$$h = 8$$

8 ft

*Solve each equation for the given variable.*

a) $2m - nx = x + 4 \;\; for\ x$

$$+nx \quad +nx$$

$$2m = x + nx + 4$$

$$-4 \qquad\qquad -4$$

$$2m - 4 = x + nx$$

$$\frac{2m-4}{(1+n)} = \frac{\cancel{(1+n)}\cdot x}{\cancel{(1+n)}}$$

$$\boxed{\frac{2m-4}{n+1} = x}$$

b) $\frac{x}{a} - 1 = \frac{y}{b} \;\; for\ x$

$$+1 \qquad +1$$

$$\cancel{a}\left(\frac{x}{\cancel{a}}\right) = \left(\frac{y}{b} + 1\right)a$$

$$\boxed{x = \frac{ay}{b} + a}$$

c) $ax + 2xy = 14 \;\; for\ y$

d) $V = \frac{1}{3}\pi r^2 h \;\; for\ h$

e) $2(A) = \cancel{2}\left[\left(\frac{f+g}{\cancel{2}}\right)h\right] \;\; for\ g$

$$\frac{2A}{h} = \frac{(f+g)\cancel{h}}{\cancel{h}}$$

$$\frac{2A}{h} = f + g$$

$$-f \qquad -f$$

$$\boxed{\frac{2A}{h} - f = g}$$

f) $2(x + a) = 4b \;\; for\ a$

# Bell Ringer

Solve for x.

① $3\left(\frac{x}{3}\right) = (10)3$

$\boxed{x = 30}$

② $5x - 7y = 12$

$+7y \quad +7y$

$\frac{5x}{5} = \frac{12+7y}{5}$

$\boxed{x = \frac{12+7y}{5}}$